

6. Determine the cross-sectional area of a 100-ft. steel cable supporting a 25,000 lb. tensile force while not exceeding the allowable tensile stress of 40,000 psi or a maximum elongation of 0.100 ft. Assume the modulus of elasticity of steel is $E = 29,000,000$ psi (assume all values are “exact” measurements).

Since there are two limiting conditions, we should check to see if they are both satisfied.

First, let's check the allowable stress condition. If the allowable stress is 50,000 psi, let's compute the minimum area to meet that state.

$$\sigma = \frac{P}{A} \Rightarrow A = \frac{P}{\sigma} \qquad A_1 = \frac{P}{\sigma} = \frac{25,000 \text{ lb.}}{40,000 \text{ psi}} = 0.625 \text{ in.}^2$$

Next, let's compute the minimum area for the maximum elongation limit.

$$\frac{P}{A} = E \frac{\delta}{L} \Rightarrow A = \frac{PL}{E\delta} \qquad A_2 = \frac{(25,000 \text{ lb.})(100 \text{ ft.})}{(29,000,000 \text{ psi})(0.100 \text{ ft.})} = 0.862 \text{ in.}^2$$

To determine which area controls the design, we should check both values in both limit states.

For the stress limit condition, compute the stress using the A_2 value

$$\sigma = \frac{P}{A} = \frac{25,000 \text{ lb.}}{0.862 \text{ in.}^2} = 29,000 \text{ psi} < 40,000 \text{ psi} \quad \text{OK}$$

For the elongation limit condition, compute the elongation using the A_1 value

$$\frac{P}{A} = E \frac{\delta}{L} \Rightarrow \delta = \frac{PL}{AE} = \frac{(25,000 \text{ lb.})(100 \text{ ft.})}{(0.625 \text{ in.}^2)(29,000,000 \text{ psi})} = 0.138 \text{ ft.} > 0.100 \text{ ft.} \quad \text{NOT OK}$$

Therefore, the value that satisfies both the stress and elongation limit conditions is

$$A = 0.862 \text{ in.}^2$$